# Individual Project 1
# Upper and Lower Riemann Sums
# Due Friday, Jan. 15

*****Read this first!** – You will need to email your professor (me) to get the specific function, interval, and error bound to use in your report. Be sure to do this early enough to get this data and complete your report. (e,g. if you email me at 11:00 PM, I probably will not get you the data until the following day.)

**Writing Style:** Since this is the first project, let us begin by considering writing style. In this assignment I will include an itemized list of the points I will be looking for in your report. I am doing this in order to help you more easily determine if you have addressed all the requirements in the assignment. Your report, however, should not just be an itemized list. It should stand on its own without referring to this assignment. A person who has not read this assignment should still be able to understand your report. I will not accept a report that is simply a list of answers.

In addition, the projects in the class will often involve a significant number of equations and other expressions using mathematical symbols. In order to include these in your reports in a professional manner, it will be necessary to use some tool to typeset your mathematical equations. The Equation Editor that is included with Microsoft Word is one good option. You should work to include your equations smoothly into your reports. Remember, mathematical symbols represent words. They can be used in sentences as if they were the words they represent.

I will be looking for the following in grading your report (see the syllabus for more details):

1. Units of measurement where they are appropriate (e.g. foot, second, meter, pound, etc.)
2. Smooth incorporation of mathematical notation
3. Complete sentences
4. Adequate explanations of your reasoning and results

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**Project Assignment:** Let $f(x)$ be a positive, continuous, increasing function on $[a, b]$; you should request a specific function and interval for your report by email. Your goal is to develop a technique to estimate the integral $\int_a^b f(x)\ dx$ within a margin of error without evaluating the integral.

1. Suppose you partition $[a, b]$ into $n$ equal parts. We will approximate the area represented by the integral by adding up areas of rectangles, one on each subinterval, that approximate the area under the curve. The width of each rectangle will be the width of the subinterval and the height will be that of some point on the curve within that subinterval. Depending on how we pick this point we will get different approximations of the area, but all of these sums of areas of rectangles are called Riemann sums. Determine the length of each subinterval of your specific interval $[a, b]$ if there are $n$ subintervals.

2. For our first type of Riemann sum, choose the height of each rectangle to be the maximum value of $f$ over the subinterval. Since $f$ is increasing, think about where this maximum value will occur. The Riemann sum defined by taking this maximum on each subinterval is called an *upper sum*. If there are $n$ subintervals we write this as $U_n$. Calculate $U_4$ for the function and interval you obtained for your report.

3. Explain why $\int_a^b f(x)\,dx \leq U_n$ for every value of $n$.

4. For the second type of Riemann sum, use the same procedure but we use the minimum value of $f$ over each subinterval. This Riemann sum is called a *lower sum* and, with $n$ subintervals, is written $L_n$. Calculate $L_4$ for your function and interval.

5. Explain why $\int_a^b f(x)\,dx \geq L_n$ for every value of $n$.

6. Now consider the difference $U_n - L_n$. Explain why this difference is non-negative.

7. Explain how you know that if either $U_n$ or $L_n$ where used as an approximation for $\int_a^b f(x)\,dx$ then the error would be less that $U_n - L_n$.

8. For a general positive, continuous, increasing function $f(x)$ simplify $U_n - L_n$ as much as possible. This should be a <u>very</u> simple expression. Explain why this expression is correct.

9. Suppose you are given your function $f(x)$ on the interval $[a, b]$ and a positive real number $\epsilon$. Explain how you can use the results so far to to estimate $\int_a^b f(x)\,dx$ with a margin of error at most $\epsilon$.

10. Find an estimate of $\int_a^b f(x)\,dx$, for your specific function and interval, that is within the error bound you received for your report.

You should print your report and turn it in by the due date.